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# An elementary introduction to the Holographic Principle

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Nowadays it is clear that Quantum Mechanics and General Relativity have been the two most important paradigms in Fundamental Physics during the last century. The year 2005 was chosen by the UNESCO as the World Year of Physics with the aim to commemorate the Einstein's miraculous year (*annus mirabilis*) 1905, where he shocked the scientific community by presenting in the volume 17 of *Annalen der Physik* a set of articles [1] about the Photoelectric Effect, the Brownian Motion and Special Relativity. In the first one Einstein set the concept of light quanta (*Lichtquanten*), which was one of the conceptual cornerstones of Quantum Mechanics, and in the last two he set the basis of the Theory of Relativity that would give rise to General Relativity several years later. Professor Galindo, to whom this article is dedicated, has devoted a great deal of his fruitful scientific career to Quantum Mechanics and General Relativity as a scholar, researcher and teacher, and from him several generations of Spanish theoretical physicists benefited. It is really a honor and a pleasure to have the opportunity to celebrate, almost simultaneously, the 70th birthday of Alberto Galindo and the beginning of the World Year of Physics 2005. We will do that here by reviewing briefly the genesis and present status of the so called *Holographic Principle*. This is because according to the opinion of many people it could bring some light on the always obscure connection between Quantum Mechanics and General Relativity, the big themes of contemporary Fundamental Physics and the academic life of Alberto Galindo.

## 1 The Generalized Second Law

One of the most intriguing predictions of General Relativity (GR) is the existence of horizons in many of its solutions. These horizons establish limits and boundaries between different sets of events determining the causal structure of space-time. Probably the most popular solution of the GR Einstein's 1915

field equations [2] is the Schwarzschild one, which is believed to represent the external metric of black-holes (BH). These bizarre objects were already predicted to exist at the Age of Enlightenment ([3] and [4]), but only after the advent of GR they have a sound theoretical support (the term BH was coined by Wheeler in 1967). The Schwarzschild metric can be generalized to the Kerr-Newman (KN) solution [5] which describes the most general stationary BH exterior and in general has a quite rich structure of horizons and even connections with multiple universes (see for instance [6] for a detailed description). The only parameters appearing in this solution are the mass  $M$ , the angular momentum  $J$  and the electric charge  $Q$  of the BH. This surprising result is known in the literature as the *no-hair theorem*. The proof for four dimensions can be found for instance in [7] or [8] (the theorem does not apply to arbitrary number of dimensions). It is quite paradoxical since it means that the final state of any system collapsing into a BH, no matter how complex the system could be, will be described by just three parameters.

In any case the area  $A$  of the event BH horizon is given in Planck units by:

$$A = 4\pi(R_+^2 + a^2), \quad (1)$$

where

$$R_+ = M \left( 1 + \sqrt{1 - \frac{Q^2 + a^2}{M^2}} \right) \quad (2)$$

and  $a = J/M$  (we are assuming  $Q^2 + a^2 \leq M^2$ ). On the horizon the angular frequency, the surface gravity and the electric potential are given respectively by:  $\Omega = 4\pi a/A$ ,  $\kappa = 4\pi(R_+ - M)/A$  and  $\Phi = 4\pi R_+ Q/A$ .

At the beginning of the seventies, the work of several people made possible to establish a set of rules concerning the state and evolution of classical BH's [9]. These rules can be stated as follows (see for instance [10]):

*Rule 0:* If the BH is stationary the surface gravity  $\kappa$  on the horizon is constant.

*Rule 1:* Under small variations of the parameters of a stationary BH we have:

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ \quad (3)$$

*Rule 2:* Given a system with an arbitrary number of BH's with area  $A_n$ , the temporal evolution is such that the variation of total horizon area  $A$  does not decreases with time:

$$dA = \sum_n A_n \geq 0 \quad (4)$$

*Rule 3:* A stationary BH with  $\kappa = 0$  is not accessible by a finite number of steps.

These rules lead to Bekenstein to suggest an analogy between *classical* BH and thermodynamics [11]. According to it a stationary BH behaves like a system in thermodynamic equilibrium so that the rules above correspond with the laws of thermodynamics. One important consequence of this analogy is that the BH has an entropy proportional to its horizon area, i. e.  $S_{BH} = CA$ . In addition Bekenstein also enunciated the so called *Generalized Second Law* (GSL). According to it whenever a system may suffer a gravitational collapse the total entropy  $S$  must be defined as the sum of the standard matter-entropy  $S_m$  plus the entropy of the BH, that could eventually appear, as defined above, i.e.  $S = S_m + S_{BH}$ . Thus the GSL states that it is this total entropy the one that is never decreasing when strong gravitational effects must be taken into account. The GSL, by assigning some entropy to the collapsed matter inside the BH, obviously solves the *no-hair theorem* paradox. However this solution is not complete since the nature of the  $\exp S_{BH}$  states given rise to this BH entropy remains absolutely unknown.

## 2 The Information Paradox

The BH thermodynamic description given by Bekenstein was suddenly supported in an impressive way through a semi-classical computation done by Hawking that showed that BH's do in fact radiate [12]. The radiation is thermal corresponding to a temperature  $T = \kappa/2\pi$  which set the constant  $C$  to one quarter so that the BH entropy is given by:

$$S_{BH} = \frac{A}{4} \quad (5)$$

One of the most important consequences of Hawking radiation is that BH's lose mass, shrink their surface and eventually disappear into a cloud of thermal energy. It can be shown that during the process of gravitational collapse of some system into a BH and its ulterior decay into radiation the GSL applies, i.e. the total entropy never decreases (in fact it increases). However BH evaporation poses a new challenge to our scarce knowledge of the quantum aspects of gravity. The problem was outlined by Hawking as a lost of unitarity in the evolution of the collapsed system (*information paradox*)[13]. It may be understood in a very simple way considering the case in which the initial state of the collapsed matter is a pure quantum state. After the Hawking evaporation of the BH we are left just with thermal radiation which is not a pure quantum state and must be described by a density matrix. Therefore the whole evolution of the system cannot be unitary.

This statement gave rise to a great controversy in the physics community during the last three decades. Defenders of orthodox quantum theory such as Coleman, Thorne, Preskill and others have proposed different mechanisms to scape from any violation of unitarity. For example it has been argued that, since the Hawking computation is only approximate (semi-classical),

subtle correlations in the radiation not taken into account, could maintain the information content of the system. This is for instance what happens if we burn a volume of the *Encyclopædia Britannica*. Correlations in the relative motion of the molecules in the produced smoke encode all the information contained in the volume. Needless to say that for all practical purposes the information is completely lost but still the evolution of the system is unitary.

Another solution considered for the *information paradox* is that, as far as the final moments of the life of the BH are determined for some unknown quantum gravity theory, it is possible that after Hawking evaporation always a remnant is left which stores the information contained in the BH. However it is difficult to see how a Planck-scale sized object could carry all the information of, for example, a collapsed star.

There are more creative solutions like the supposition that the BH give rise to a new universe to which the information flows. In this way unitarity is preserved in the whole set of universes but it is apparently lost for an observer outside of the BH.

Finally even Hawking seems to have changed his mind concerning this issue and recently he has announced [14] a new mechanism that could avoid unitarity violations in BH evaporation (the details are yet unpublished) .

### 3 Entropy Bounds

In 1981 Bekenstein proposed the *Universal Entropy Bound* (UEB) [15] which states that the entropy  $S$  of a complete physical system in asymptotically flat  $D = 4$  space-time, whose total mass-energy is  $E$ , and which fits inside a sphere of radius  $R$ , is necessarily bounded from above:

$$S \leq 2\pi ER \quad (6)$$

This bound can be obtained through a *gedanken* experiment in which a weakly self-gravitating object with some entropy content is left with the lesser possible energy at a BH horizon. Apparently the bound above follows if one wants to avoid any violation of the GSL. The UEB is important because it is an attempt to set limits on the entropy of system which is characterized by physical parameters such as energy and size. This kind of bound could be relevant, at least in principle, for information theory, both classical and quantum (see [16] for a recent and very complete review). Since the seminal Shannon's works the main topic of information theory has been information transport, or in other words, channel capacity, noise, redundancy, cryptography and many other things which have to do with information communication. However the UEB refers to information storage capacity. Usually this issue is contemplated under the point of view of the smaller physical system capable to store one single bit of information like molecules, atoms, photon polarization etc. The point of view of the UEB is completely different since it offers a more holistic

kind of bound which applies to the whole memory system independently of its microscopic structure. Note also that the bound does not contain (in standard units) the Newton constant at all (it refers only to weakly self-gravitating systems but this is the case of most of lab and astrophysical systems around us). On the other hand, from the point of view of present applications, the UEB is far from being of any practical interest. For example for a standard music compact disk the UEB set a maximum of storage capacity of about  $10^{68}$  bits but present technology make possible to put *only*  $10^{10}$  bits on it. However, as a matter of principle, the UEB defines a new fundamental relation between energy, size and information.

Another kind of bound on the entropy (information) content of non-BH objects was proposed by Susskind in 1995 [17]. According to it the maximum entropy  $S_m$  of a system that can be enclosed by a spherical surface of area  $A$  is given by:

$$S_m \leq \frac{A}{4} \quad (7)$$

This bound is known as the *Spherical Entropy Bound* and it requires the space-time to be asymptotically flat. The proper definition of  $A$  also assumes that the system has spherical symmetry or that it is weakly gravitating. The bound is motivated by considering another *gedanken* experiment called the Susskind process. One consider a system of mass  $E$  inside an area  $A$  which is smaller than the mass  $M$  that would produce a BH if fitted in the same area  $A$ . Now we add an infalling spherical shell of mass  $M - E$  in order to collapse the system. Then applying the GSL to the process the bound follows.

The *Spherical Entropy Bound* can be derived, under some conditions, from the UEB so in the cases where both can be applied the former is weaker than the latter. However the *Spherical Entropy Bound* is much more appropriate to introduce the main ideas underlaying the *Holographic Principle*. Note that this bound is telling us that the maximum information that a system can store scales basically with the area of its external surface. This is in clear contradiction with our normal experience according to which the information capacity scales with the volume.

The *Spherical Entropy Bound* can also be extended to a much more general bound called the *Space-like Entropy Bound*. The formulation of this bound is the following [18]: Let be a compact portion of equal time spacial hypersurface in space-time with volume  $V$  and boundary  $B$  of area  $A(B)$ , then the total entropy inside the volume  $V$  is bounded by:

$$S(V) \leq \frac{A(B)}{4} \quad (8)$$

This bound seem to be the natural generalization of the *Spherical Entropy Bound*. It works in many systems but it is easy to find counter examples where it does not apply, meanly in cosmological scenarios or for strongly gravitational systems (see [18] for a detailed discussion). The failure of this

bound lead Bousso to propose a suitable generalization of this bound which is known as the *Covariant Entropy Bound* that will be discussed latter.

## 4 The 't Hooft Holographic Principle

After almost one decade of lonely efforts outside of the main stream in theoretical physics 't Hooft [19] (followed by Susskind [17]) presented his *Holographic Principle*. This principle changes radically our thinking about the counting of degrees of freedom of physical systems and the way in which the entropy or information content is stored.

There is no any well established enunciate of this principle but according to the ideas of 't Hooft and Susskind one possible preliminary formulation of the *Holographic Principle* could be:

The full physical description some given region  $R$ , in an  $D$  dimensional universe, with  $D - 1$  dimensional boundary  $B = \partial R$ , can be reflected in processes taken place in  $B$ .

Clearly the above formulation is too vague to be of practical interest but still the *Holographic Principle* is regarded as a major clue in the search for the solution for the Quantum Mechanics versus GR conundrum. In particular any fundamental theory should incorporate this counterintuitive result.

In fact we have already an example where the *Holographic Principle* could be taking place, namely the Maldacena conjecture known as the *AdS/CFT correspondence* [20]. According to it, for some given settings, the physics of a string theory of type IIB, defined on an  $AdS_5 \times S^5$  space, is equivalent to the physics of a maximally supersymmetric Super Yang-Mills  $U(N)$  theory defined on the boundary of the  $AdS_5$  space. Even in the absent of a real proof, the Maldacena conjecture has passed successfully a great number of checks and it is generally believed to be true.

In spite of the fact that the *Holographic Principle* is not yet defined in a precise way, it is clear that, from a fundamental point of view, the entropy bounds discussed in the previous section are likely to be a more or less straightforward consequence on this principle. In any case, in the absence of a well formulated *Holographic Principle*, entropy bounds can be extremely useful as heuristic tools for the task of clarify the apparent contradictions between Quantum Mechanics and GR.

## 5 The Covariant Entropy Bound

In order to solve the difficulties found in the *Space-like Entropy Bound*, Bousso proposed the *Covariant Entropy Bound* (CEB) [21, 18]. The bound can be stated as follows: Let  $B$  be any spatial  $D - 2$  dimensional hypersurface with area (volume)  $A(B)$ . A  $D - 1$  dimensional hypersurface  $L$  is called a light-sheet of  $B$  if it is generated by a congruence of null geodesics beginning at  $B$ ,

extend orthogonally from  $B$  and has negative expansion. Now we define  $S$  as the entropy of any matter *illuminated* by the  $B$ 's light sheet. Then

$$S(L) \leq \frac{A(B)}{4} \quad (9)$$

In order to clarify the meaning of the CEB note that for any point in  $B$  it is possible to construct four light rays (branches). Two of them go to the future and two go to the past. On any of these branches, a ray, together with its neighbors, defines a positive or a negative expansion (rays converging or diverging). The  $L$  set considered in the above formulation of the CEB is the one corresponding to future going converging congruence. The rays so defined may end at the tip of a cone (for spherical symmetry) or more generally on a caustic. After this the rays will start to diverge but this region is considered to be outside of  $L$ . The entropy of the matter traversed by this set of rays before they reach the caustic is the one bounded according to the Bousso's covariant entropy bound.

The CEB can successfully solve the difficulties found by the *Space-like Entropy Bound*, in particular in cosmological and strong gravitating scenarios. In fact it holds in a wide range of situations and no physically realistic counter example has been found so far. A logical consequence of this is to consider the CEB as a hint for more fundamental law of nature. According to Bousso the *Covariant Holographic Principle* could be stated (using the previous notation) as follows: The fundamental theory underlying Quantum Mechanics and GR should be such that the matter and the geometry illuminated by the convergent rays starting from  $B$  have a number of independent states  $\mathcal{N}(L(B))$  which is bounded by:

$$\mathcal{N}(L(B)) \leq e^{A(B)/4} \quad (10)$$

If the fundamental theory contains Quantum Mechanics in its present form, the number of (quantum) states  $\mathcal{N}$  is just the dimension of the Hilbert Space but this does not have to be necessarily the case. In the language of information theory the number of bits times  $\ln 2$  involved in the description of  $L(B)$  must be bounded by  $A(B)/4$ .

There is also a stronger version of the CEB which was proposed in [22] known as the *Generalized Covariant Entropy Bound* (GCEB). In this version the light rays in  $L$  are allowed to stop before they reach the caustic and in this way they define a new surface  $B'$  of area  $A(B')$ . Then according to the GCEB we have:

$$S(L) \leq \frac{A(B) - A(B')}{4} \quad (11)$$

Obviously the GCEB reduces to the CEB for the particular case  $A(B') = 0$ . In [22] and [23] some different sets of necessary conditions for the energy and entropy matter content are shown to guarantee the validity of the GCEB and it has being shown that the UEB can be obtained from the GCEB [24]. It has also been argued that CEB must be modified in some way in order to include Hawking radiation [25].

## 6 Discussion

Independently of the precise formulation of the *Holographic Principle* or its possible consequences, for example the entropy bounds, it is apparent that there are many indications that point towards the validity of some law of a similar kind. Therefore this hypothetical law should be encoded in any fundamental theory that may reconcile Quantum Mechanics and GR. However, even in the absence of such a theory, there are some important conclusions that can be drawn from the *Holographic Principle*.

The first one is that Quantum Field Theory (QFT) does not work when strong gravitational effects are present. In order to see why this is the case we can consider a QFT defined in a finite volume  $V$  (to avoid infrared divergencies). Then the divergencies are ultraviolet and typically they are regulated by means of an energy cutoff (or equivalently by introducing some minimal distance). In any case the number of degrees of freedom scales with the volume  $V$  of the system and not with the external area as the *Holographic Principle* seems to suggest. The reason for this is that even when the theory is regularized by cutting the high energy modes, there remain an enormous number of field configurations that are gravitationally unstable and would collapse into a BH. By removing all of these configurations we are left with a much less number of states that would scale with the external area of the system in a way compatible with the *Holographic Principle*.

On the other hand many people consider Superstring Theory as the most sound candidate for a quantum theory of gravity. It has been argued that Superstring Theory violates the *Holographic Principle* since the number of states scales also with the volume of the system. However this is true only at the perturbative level. When non-perturbative effects are taken into account in Superstring Theory one has to deal with the so called M-theory. The non-perturbative regime can be reached in some cases by means of some duality transformation from the perturbative one but the physics turn out to be completely different in general. In particular strings give rise to new bound states (D-branes) in such a way that the counting of degrees of freedom may change drastically from the perturbative regime. Moreover it has been shown that the perturbation series breaks down before the holographic bounds are reached. In fact the *AdS/CFT Correspondence* and the successful computation of the entropy in some kind of BH, done in the framework of M-Theory by counting microscopic states [26], seems to suggest that M-theory could be compatible with the *Holographic Principle*.

In any case one can consider the *Holographic Principle* as a necessary ingredient of any fundamental theory and use it in order to make predictions even if the fundamental theory is far from being established. Finally, as in any other branch of physics, confrontation with the experiment will be the final test for this principle and if the general idea is correct it can play an important role in the future.



To end it is important to stress that in this paper we have not considered the possibility of having a non-zero cosmological constant. However recent data, coming from the cosmic microwave background radiation, distant supernovae, and the spectrum of the density fluctuations, strongly suggest that the cosmological constant is different from zero. If this is the case almost all the points mentioned in this article should be reconsidered even if for many applications the cosmological constant can be safely neglected. Work is in progress in this direction.

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